Refinement of Non-Linear Magnetic Models via a Finite Element Subproblem Method

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Abstract— Model refinements of non-linear magnetic circuits are performed via a finite element subproblem method. A complete problem is split into subproblems to allow a progression from 1-D to 3-D including linear to non-linear model corrections. Its solution is then expressed as the sum of the subproblem solutions supported by different meshes. A convenient and robust correction procedure is proposed allowing independent overlapping meshes for both source and reaction fields. It simplifies both meshing and solving processes, and quantifies the gain given by each refinement on both local fields and global quantities.

I. Introduction

The perturbation of finite element (FE) solutions provides clear advantages in repetitive analyses and helps improving the solution accuracy [1]-[6]. It allows to benefit from previous computations instead of starting a new complete FE solution for any variation of geometrical or physical data. It also allows different problem-adapted meshes and computational efficiency due to the reduced size of each subproblem.

A FE subproblem method (SPM) is herein developed for coupling solutions of various dimensions, starting from simplified models, based on ideal flux tubes defining 1-D models, that evolve towards 2-D and 3-D accurate models, allowing leakage flux and end effects. Progressions from linear to non-linear models are aimed to be performed at any step, which extends the method proposed in [3]-[6]. A convenient and robust correction procedure is proposed here. It combines any changes, via volume sources (VSs) and surfaces sources (SSs), with possible superpositions in single correction steps. It allows independent overlapping meshes for both source and reaction fields, which simplifies the meshing procedure.

The developments are performed for the magnetic vector potential FE magnetostatic formulation, paying special attention to the proper discretization of the constraints involved in each SP. The method will be illustrated and validated on test problems.

II. Progressive Magnetic Subproblems

A. Sequence of Subproblems

General 2-D and 3-D non-linear models are proposed to be split into sequences of SPs, some of lower dimensions, i.e. 1-D and 2-D models, and others for adequate corrections of various types. Non-linear corrections are aimed to be allowed at any level of this sequence. The SP solutions are to be added to give the complete solution. This offers a way to perform model refinements, with a direct access to each correction, usually of useful physical meaning.

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Each SP is defined in its own domain. At the discrete level, this aims to decrease the problem complexity and to allow distinct meshes with suitable refinements and possible domain overlapping, each SP having to approximate at best its contribution to the complete solution.

B. Canonical magnetic problem

A canonical magnetostatic problem p is defined in a domain Ω_p, with boundary $\partial\Omega_p = \Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p}$. Subscript p refers to the associated problem p. The equations, material relation, boundary conditions (BCs) and interface conditions (ICs) of problem p are

$$\mathrm{curl}\, \boldsymbol{h}_p = \boldsymbol{j}_p\,, \quad \mathrm{div}\, \boldsymbol{b}_p = 0\,, \quad \boldsymbol{h}_p = \mu_p^{-1}\, \boldsymbol{b}_p + \boldsymbol{h}_{s,p}\,, \quad \text{(1a-b-c)}$$

$$\boldsymbol{n} \times \boldsymbol{h}_p|_{\Gamma_{h,p}} = 0\,, \quad \boldsymbol{n} \cdot \boldsymbol{b}_p|_{\Gamma_{b,p}} = 0\,, \quad \text{(1d-e)}$$

$$[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p} = \boldsymbol{j}_{f,p}, \quad [\boldsymbol{n} \cdot \boldsymbol{b}_p]_{\gamma_p} = \boldsymbol{b}_{f,p}, \quad \text{(1f-g)}$$

where $\boldsymbol{h}_p$ is the magnetic field, $\boldsymbol{b}_p$ is the magnetic flux density, $\boldsymbol{j}_p$ is the prescribed current density, μ_p is the magnetic permeability and $\boldsymbol{n}$ is the unit normal exterior to Ω_p. The notation $[\,\cdot\,]_\gamma = \cdot|_{\gamma^+} - \cdot|_{\gamma^-}$ expresses the discontinuity of a quantity through any interface γ (with sides γ^+ and γ^-) in Ω_p, which is allowed to be non-zero.

The field $\boldsymbol{h}_{s,p}$ in (1) is a VS, usually used for fixing a remnant induction. With the SPM, $\boldsymbol{h}_{s,p}$ is also used for expressing changes of permeability, e.g. for added regions and non-linear changes. For a change of permeability of a region, from μ_q for problem q to μ_p for problem p, the VS $\boldsymbol{h}_{s,p}$ in this region is

$$\boldsymbol{h}_{s,p} = (\mu_p^{-1} - \mu_q^{-1})\, \boldsymbol{b}_q\,, \quad (2)$$

for the total field to be related by $\boldsymbol{h}_q + \boldsymbol{h}_p = \mu_p^{-1}\,(\boldsymbol{b}_q + \boldsymbol{b}_p)$.

The surface fields $\boldsymbol{j}_{f,p}$ and $\boldsymbol{b}_{f,p}$ in (1f-g) are generally zero to define classical ICs for the fields. If nonzero, they define possible SSs. This is the case when some field traces in a previous problem q have been forced to be discontinuous, e.g. for neglecting leakage fluxes and reducing the problem to a lower dimension [2]-[6]. The continuity has to be recovered after a correction via a problem p. The SSs in problem p are thus to be fixed as the opposite of the trace solution of problem q.

Each problem p is constrained via the so defined VSs and SSs from parts of the solutions of other problems. This offers a wide variety of possible corrections [2]-[6], that welcome linear to non-linear changes as well.

III. Various Possible Problem Splittings

For a typical magnetic circuit, e.g. an electromagnet, the SP procedure commonly splits the problem into 3 SPs (Fig. 1): (1) the magnetic region and the air gaps considered as an ideal flux tube (with possible start from 1-D models [4]-[5]), (2) the stranded inductor alone, and (3) the consideration of the leakage flux via a SS $\boldsymbol{j}_{f,3}$ on the flux tube boundary, simultaneously with the change of permeability due to the addition of the magnetic region in the inductor source field [6]. In this way, steps 2 and 3 are based on totally independent meshes; step 1 uses a portion of mesh 3.

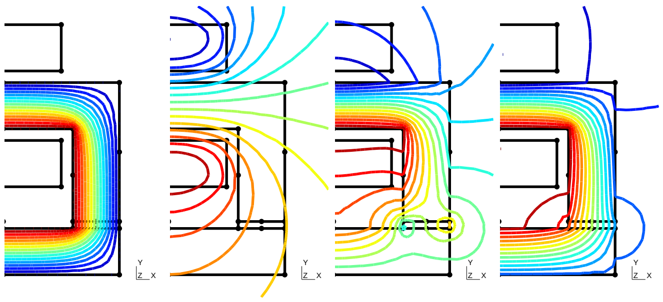

Fig. 1. Field lines in the ideal flux tube ($\boldsymbol{b}_1$, $\mu_{r,core} = 100$), for the inductor alone ($\boldsymbol{b}_2$), for the leakage flux ($\boldsymbol{b}_3$) and for the total field ($\boldsymbol{b}$) (*left to right*).

It is herein proposed to allow changes from linear to non-linear material properties in the correction SPM. An initially linear μ_q can change to a non-linear μ_p to be expressed as a function of the total magnetic flux density. The resulting VS (2) supported by the non-linear region is

$$\boldsymbol{h}_{s,p} = (\mu_p^{-1}(\boldsymbol{b}_q + \boldsymbol{b}_p) - \mu_q^{-1})\, \boldsymbol{b}_q \,. \qquad (3)$$

At the discrete level, the source quantity $\boldsymbol{b}_q = \mathrm{curl}\, \boldsymbol{a}_q$, initially given in mesh q, is projected in the mesh p [6], limited to the non-linear region. A classical non-linear iterative process has then to lead to the convergence of $\boldsymbol{b}_p = \mathrm{curl}\, \boldsymbol{a}_p$. This solution corrects the flux linkages of the inductors, and consequently their reluctances. It will be shown that the reluctance correction can be accurately calculated via an integration limited to the non-linear region, with no need to integrate the flux density linked to the inductor, part of a different mesh.

Various combinations of problem splitting will be studied, discussed and validated in the extended paper, combining any of the following steps in various orders: inductor(s) alone, perfect magnetic materials (infinite permeability) or saturated materials, linear or non-linear ideal flux tubes (from 1-D to 3-D), linear or non-linear real tubes with leakage flux (from 2-D to 3-D). The results of a two-step SPM from linear to non-linear problems are shown in Figs. 2 and 3 for high and low reluctance circuits, illustrating the way the correction fields behave (the first step is actually the combination of other steps, considering the inductor alone and the added linear magnetic material). An initial estimation of μ_1, e.g. from a 1-D linear model, can help the non-linear correction process to reduce the correction (Fig. 2). Another correction, from an ideal flux tube to a non-linear one with leakage flux, is illustrated in Fig. 4. The developed combinations will be shown to help for a better understanding of magnetic circuit behaviors, regarding non-linear properties, leakage flux, and 2-D and 3-D effects.

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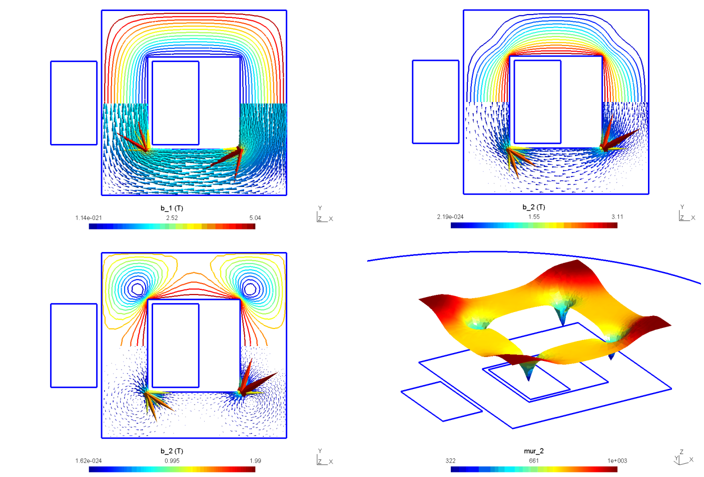

Fig. 2. Field lines and magnetic flux density for the linear model ($\boldsymbol{b}_1$, $\mu_{r,1}$=1000, *top left*) and its non-linear correction ($\boldsymbol{b}_2$, *top right*); another non-linear correction ($\boldsymbol{b}_2$, from $\mu_{r,1}$=780, *top right*); final relative permeability ($\mu_{r,2}$, *bottom right*).

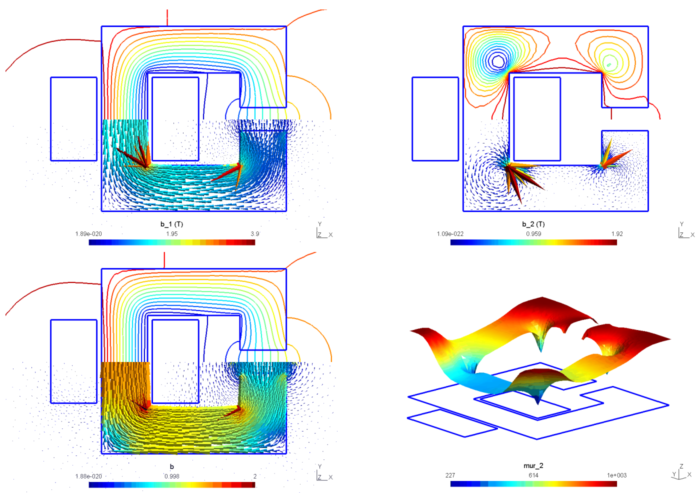

Fig. 3. Field lines and magnetic flux density for the linear model ($\boldsymbol{b}_1$, $\mu_{r,1}$=1000, *top left*) and its non-linear correction ($\boldsymbol{b}_2$, *top right*), and for the total solution ($\boldsymbol{b}_1$+$\boldsymbol{b}_2$, *bottom left*); relative permeability ($\mu_{r,2}$, *bottom right*).

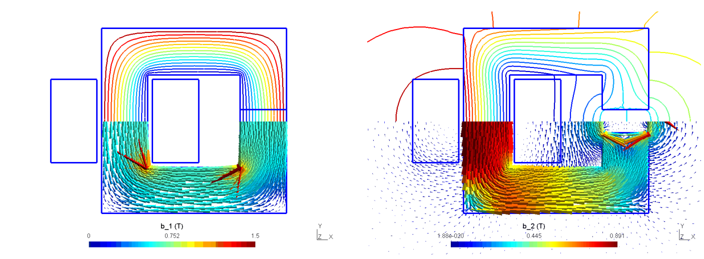

Fig. 4. Field lines and magnetic flux density for the ideal flux tube model ($\boldsymbol{b}_1$, *left*) and for the non-linear correction with leakage flux ($\boldsymbol{b}_2$, *right*).